

A guide to breach prediction

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Executive summary

Earthen embankments that are categorised as large (15m or greater in height) number in the tens of thousands globally. Embankment dam risk assessment is a vital measure that has been adopted throughout the industry to assess the potential impact that catastrophic dam failures can carry. A critical part of this assessment is the prediction of the breach process, which will determine the reservoir outflow hydrograph. This is crucial for the following stage of flood routing, which aids in flood risk assessment, evacuation planning and land-use planning.

This report provides details of the breach prediction methods available to users, ranging from simple parametric equations to complex multi-dimensional erosion models. These are commonly divided into three categories; parametric models, semi-physically based models and physically based models.

Parametric models, such as Froehlich (2016a, b), Xu & Zhang (2009) and Von Thun & Gillette (1990), allow breach geometry, formation time and peak outflow to be estimated through the regression analysis of historical dam failure data. These have advantages in their ease and speed of use, but were found to have great uncertainty in their application and are therefore not typically suitable for high risk applications, where uncertainties will have a large impact. Appropriate applications include initial appraisal-level breach modelling and the study of low-risk scenarios where uncertainties will have a minimal impact.

Semi-physically based models, such as HEC-RAS, take breach geometry and formation time, or soil erosion rates, as input values to produce a breach hydrograph. No physical processes are modelled; rather the flow of water through the use-defined breach is calculated using simple fluid dynamic equations, such as weir and orifice flow. These provide no improvement in the accuracy of predicting a breach over parametric models, but improve on the process of converting these results into outflow hydrographs, which may be required for further use.

Physically based models, such as EMBREA, DL Breach and WinDAM consider the complex geotechnical, structural and hydraulic behaviour of an embankment dam and its impounded reservoir. While generally more time-consuming than parametric and semi-physical models, physical models tend to provide results with a greater certainty and accuracy. These, as a whole, are suitable for high risk breach scenarios, where accuracy and reliability are critical in providing results within the acceptable bounds of uncertainty.

The descriptions and recommendations of breach prediction methods in this report are intended to guide users towards the most appropriate breach model for a given scenario. A recommended approach to choosing a model type (parametric, physical etc.) is given, taking into consideration the type of analysis and associated risks, amongst other factors.

It was concluded that, while parametric models have tended to be used by industry in the past, technological advancements and practical field testing have allowed rigorous physically based models methods to become more feasible. Further developments are likely to reduce the reliance of physical models on simplifications and improve their accuracy and usability.

1. Introduction

Earthen embankment dams are a crucial part of the World's infrastructure and provide electricity, irrigation, flood control and water supply to millions of people. Embankment dam risk assessment is a measure that has been adopted to help mitigate the catastrophic impact of dam break flooding. While a very unlikely occurrence, extreme weather events, critical construction deficiencies and other unforeseen circumstances have led to many major dam failures in the last two centuries. These have resulted in significant loss of life, as well as economic, social and environmental damage.

The physical breaching of a dam is a difficult phenomenon to predict and model, due to the complex interaction of hydraulic, structural and geotechnical properties. As such, many different methods with varying complexities have been developed to allow the ability to estimate the geometry and outflow of an embankment breach.

This report aims to put the variety of methods available into context, present the advantages and disadvantages of each, and ultimately aid individuals in selecting the most appropriate breach prediction method for a given scenario. The methods available range from simple empirically derived equations to complex, multi-dimensional physically based erosion models.

In this report, four breach prediction method types will be described; Rules of Thumb, Parametric models, Semi-physically based models and physically based models. The latter is often subdivided into two types; simple physically based models and more complex physically based models. Further detail and recommendations will be given on those physically based models that are publicly or commercially available. These are typically simplified to a degree, as the more complex models are computationally demanding and often lack the flexibility required for industry use.

Figures 1 & 2 show two notable embankment dam breaches that have occurred in recent history.



Figure 1 - Failure of Teton Dam, Idaho, on 6 June 1976.

This has been recorded as the highest dam to fail, and is one of the very few to have been photographed during the breaching process. The resulting flood caused 11 fatalities and estimated damages in the hundreds of millions of dollars. The failure mode was internal erosion (Barnes, 1992). Image Source: Mrs Eunice Olson (1976).



Figure 2 - Aftermath of the failure of Baldwin Hills Dam, California, on 14 December 1963.

The resulting flood caused 5 fatalities and estimated damages of \$11 million. The failure mode was internal erosion (Barnes, 1992). Image Source: L.A. Times (1963).

2. Initiating failure modes and erosion

While many dam failure mechanisms occur in the real world, the primary two considered in embankment breach modelling are overtopping / overflow and internal erosion. According to Foster (2000), these accounted for 48% and 46% respectively of all embankment dam failures up to 1986 (see Figure 3 for further details).

An overtopping failure may occur when an unsteady flow, such as a wave, travels over the crest of a dam. An overflow failure is similar, but refers to steady flow, generally the gradual overflow of water over the dam crest, due to inadequate spillway / outflow capacity. Both will cause water to cascade across the downstream face of the dam and cause erosion, which may in turn cause the dam to fail. An overtopping failure can be caused by earthquakes or landslides producing large waves, meanwhile overflow failures, which are far more common, are often caused by large or unexpected hydrologic events, or as the result of serious spillway or gate malfunctions. It is worth noting that both are often referred to collectively in breach modelling practice as overtopping.

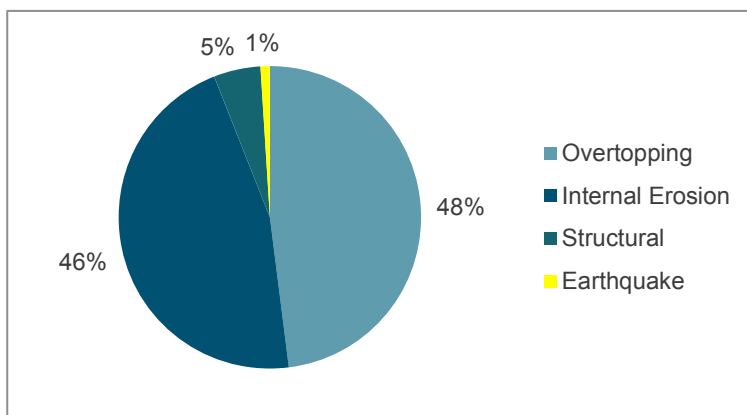


Figure 3 - Failure statistics for large embankment dams up to 1986, excluding those constructed in Japan pre-1930 and China (Foster et al, 2000)

Two important types of embankment erosion can occur during an overtopping / overflow failure; Surface and Head cut erosion. Surface erosion will typically occur in dams formed of non-cohesive material (i.e. gravel, sand) and is exhibited in the embankment shown in Figure 4. Head cut erosion will typically take place in cohesive materials (i.e. clay, clayey silts) and is exhibited in the embankment shown in Figure 5. It is important to distinguish between the two and to select the appropriate mode when modelling a breach using a physically based model, as the resulting outflow hydrograph will be very different for the two (An embankment failing under head cut erosion will typically show a greater time to peak outflow than that of surface erosion).



Figure 4 - Overflow failure and surface breach progression of a non-cohesive, rock fill embankment with moraine core (IMPACT Project) Image source: Morris (2011)

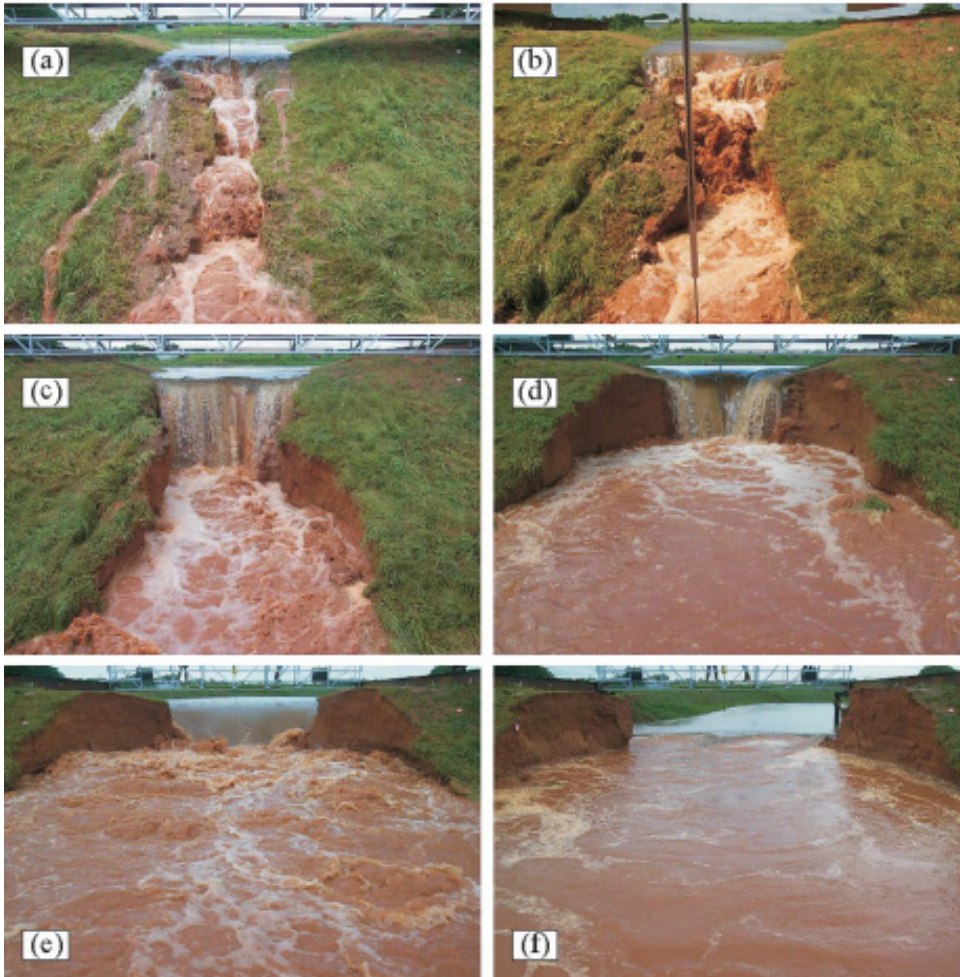


Figure 5 - Overflow failure and head cut breach progression of a cohesive embankment. Image source: Hanson et al (2005a)

Internal erosion is exhibited in the embankment shown in Figure 6 and can be caused by many factors, including foundation defects, poor construction methods, porous dam materials and the failure of cover and protection layers. This process can be split into two stages. During the first, surface erosion will take place inside the embankment, progressively allowing an increasing flow of water through the dam and increasing the size of the breach. The second stage begins when roof collapse occurs (Figure 6e), at which point the embankment breach will start to behave similarly to that of an overtopping failure.

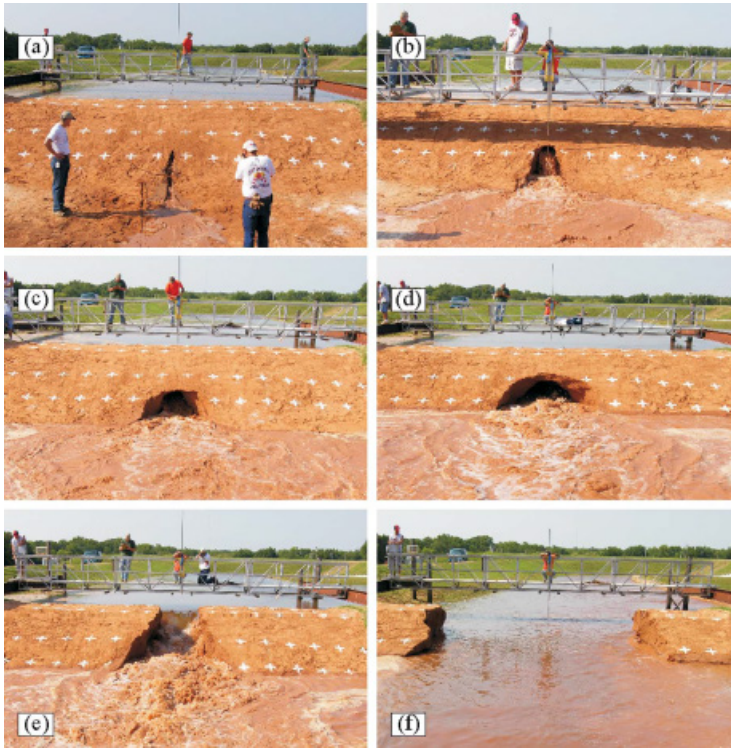


Figure 6 - Internal erosion failure and breach progression of an embankment. Image source: Hanson et al (2010)

During an overtopping / overflow failure and the second stage of an internal erosion failure, block failure may also occur. This process is governed by slope stability and refers to large chunks of material breaking away from the embankment at the sides of the breach (Figure 7).



Figure 7 - Block Failure of an embankment. Image source: Morris (2011)

3. Breach prediction methods - rules of thumb

The simplest approach to predicting a dam breach, albeit one with potentially a very high uncertainty, is guessing values based on historical evidence. Table 1, below, provides estimate value ranges from several organisations, which may be used for flood risk modelling and assessment. It is recommended that these values should be accompanied by a sensitivity analyses where possible, to account for the wide ranges given and to find the worst case scenario.

Table 1: Range of estimated values for breach characteristics. Adapted from Brunner (2014)

Source	Breach formation time, t_f (hr)	Average breach width, \bar{B} (m)	Side Slopes, z (h: v)
USACE (1980)	0.5 to 4.0	$(0.5 \text{ to } 3.0)h_d$	0 to 1.0
FERC	0.1 to 1.0	$(1.0 \text{ to } 5.0)h_d$	0 to 1.0
NWS	0.1 to 1.0	$(2.0 \text{ to } 5.0)h_d$	0 to 1.0
USACE (2007)	0.1 to 4.0	$(0.5 \text{ to } 5.0)h_d$	0 to 1.0

Where: h_d = Height of dam.

4. Breach prediction methods - parametric models

4.1. Overview

Parametric models offer simple and accessible methods to estimate the characteristics of a dam breach. Historic dam failure data is collected together and statistically analysed using regression methods. The result is a set of parametric equations describing breaching parameters as a function of simple dam or reservoir properties, typically one or several of the following; breach width, depth, side slope angle, formation time and peak outflow. An overview of the methods available and their respective equations is given in appendix B.

The main advantage of this approach is the speed, ease of use and the reduction of costs associated with using some of the more advanced physically based models. Using simple equations to determine parameters requires fewer input parameters, whilst still providing estimate values for individuals to further use. The input parameters typically include one or several of the following; Volume of water above final breach bottom (V_w); Total volume of reservoir (V_r); Height of water above final breach bottom (h_w); Height of dam (h_d) and height of breach (h_b).

There are, however, several disadvantages to these approaches that can outweigh their simplicity. The first is the lack of sufficient historical data to accurately predict breach parameters. More recently published works, such as Froehlich (2016a) and Xu & Zhang (2009), compared 111 and 182 failure cases respectively, some dating as far back as the 19th Century. Many of these are poorly documented and do not provide accurate values for peak outflow and breach geometry. Xu & Zhang (2009) only published the details of 75 dams in their final paper, due to missing or unreliable data associated with the remaining 107. Dam failure times, in particular, are hard to come by, with little over 1 in 3 of the dam failures given by Froehlich (2016a) containing this data. The majority of the failures documented across the different methods also have a height of below 15m (Wahl, 1998). As such, applying this historical information as a general rule for dam breaches, especially those that are larger, can bring about significant uncertainties.

The second disadvantage, with the partial exception of Von Thun & Gillette (1990), Walder & O'Connor (1997) and Xu & Zhang (2009), is that these methods do not take into account important factors such as dam construction, mode of failure and material properties, the most critical of which is soil erodibility, which has been proven to play a significant role in the breach process (Xu & Zhang, 2009). Many of these factors are not available in the context of historic failures, adding to the uncertainties present in parametric methods. For example, many of these equations would provide the same estimated breach geometry values for two identically shaped, but completely differently constructed, dams.

4.2. Details of specific equations

There are several important things to note regarding some of the methods listed in appendix B. Firstly, Von Thun & Gillette (1990) present two equations describing the time to failure for highly erodible and erosion resistant dams respectively. These, according to the original authored paper, should be viewed as upper and lower bounds for the value to be taken.

Walder & O'Connor (1997) provide three regression equations, respectively a function of V_w , h_w and the product of the two ($h_w V_w$), with each containing the variables a and b , which are dependent on the formation of the dam. It was concluded that there was not a clearly advantageous choice between the different relationships, so only the latter of the three is listed in appendix B.

The equations displayed from Xu & Zhang (2009) are, according to the author, the best-simplified prediction models. These use fewer parameters and take less consideration of dam type and failure mode, and so are particularly useful when limited field information is available.

Pierce et al. (2010) performed various regression methods to produce five equations relating the peak outflow of a breach to either the height, h_w or volume, V_w of water behind the breach. It was found by the authors that a linear relationship with the product ($h_w V_w$) and a linear multiple regression relationship with h_w and V_w created the two equations with the highest accuracy, and so only these are given in appendix B.

4.3. Existing comparisons of equations

Wahl (2004) provided a detailed analysis of the differences between some of the parametric methods available at the time. These included USBR (1982, 1988), Von Thun & Gillette (1990) and Froehlich (1995a, b).

He found that predictions of breach dimensions typically had uncertainties of $\pm 1/3$ order of magnitude, predictions of breach formation time had uncertainties of ± 1 order of magnitude and predictions of peak flow had uncertainties of between ± 0.5 to ± 1 order of magnitude, with Froehlich (1995a, b) being an exception with a lower magnitude of $\pm 1/3$. Overall, Froehlich (1995a, b) represented the model with the lowest overall uncertainty against measured or observed results. It was concluded by Wahl that the high uncertainties present in parametric models meant significant engineering judgement should be exercised when interpreting these results.

An individual analysis by Zhong et al (2016) compared the models of USBR (1982, 1988), Froehlich (1995a, b) and Xu & Zhang (2009) as part of a wider analysis of physically based breach models. It was claimed that the latter gave the lowest errors when predicting breach parameters, in comparison to recorded historical data, meanwhile the former provided the least accurate results.

4.4. Example datasets and application of methods

To provide a quick comparison, six of the more extensive methods outlined in appendix B were applied to the recent breaching of three different dams, in order to estimate the average breach width and formation time. The data was taken from Froehlich (2016a) and comprises of three dam failure cases not included in the development of any of the remaining parametric models.

Testalinden dam, British Columbia, Canada, was a small homogeneous earthen embankment which failed due to floodwater overflow in 2010 (the aftermath of which is shown in figure 8). Big Bay Lake dam, MS, U.S. was a large homogeneous earthen embankment which failed due to internal erosion in 2004. Delhi dam, IA, U.S. was a large core-walled earthen embankment which failed due to floodwater overflow in 2010.

As the erodibility of the dams is unknown, values obtained assuming high and low erodibility will be taken as the upper and lower bounds respectively for Xu & Zhang (2009) and Von Thun & Gillette (1990). The relevant input data is given in table 2 and the resulting values calculated using the parametric methods are given in table 3.

Table 2: Recorded data for three historic dam failure cases

Dam case	Recorded Input Data					Recorded Output Data	
	$V_w (\times 10^6 m^3)$	$V_r (\times 10^6 m^3)$	$h_w (m)$	$h_b (m)$	$h_d (m)$	$\bar{B} (m)$	$t_f (hr)$
Testalinden	0.02	0.02a	2.10b	2.10	2.10b	5.15	0.50
Big Bay Lake	17.5	17.5a	13.6	14.0	14.0b	83.2	0.92
Delhi	12.2	12.2a	11.2	11.0	11.0b	68.6	1.75

^a Assumed to be equal to the volume of water above the final breach bottom

^b Assumed to be equal to the height of the breach

Table 3: Comparison of parametric models against recorded data

Dam case	Breach Parameter	Recorded data	Parametric model prediction					
			M & L-M (1984)	USB R (1988)	V T & G (1990)	Froehlich (1995a,b)	Froehlich (2008)	Xu & Zhang (2009)
Testalinden	$\bar{B} (m)$	5.15	-	6.30	11.35	6.91	9.53	6.49 – 12.50
	$t_f (hr)$	0.50	0.09	0.07	0.03 – 1.35	0.25	0.38	0.25 – 1.37
Big Bay Lake	$\bar{B} (m)$	83.2	-	40.80	88.90	61.88	70.10	33.65 – 64.78
	$t_f (hr)$	0.92	1.05	0.45	0.20 – 1.63	1.63	1.67	1.43 – 7.73
Delhi	$\bar{B} (m)$	68.6	-	33.60	70.70	73.73	80.80	47.92 – 92.25
	$t_f (hr)$	1.75	0.90	0.37	0.17 – 1.58	1.67	1.78	1.34 – 7.24

There are several conclusions that may be drawn even from this limited comparison. The first, and most critical, is that these methods will produce a very wide range of values when estimating breach parameters, especially when considering the uncertainty of soil erodibility.

In terms of overall performance, the Froehlich (1995a) (2008) and Von Thun & Gillette (1990) models appeared to provide the most accurate results. The USBR (1988) model tended to underestimate values to a significant degree and values for failure time given by MacDonald & Langridge – Monopolis (1984) only proved reasonably accurate for one of three cases. Xu & Zhang's (2009) model is highly dependent on soil erodibility, as can be seen from the estimated value ranges, and so it is difficult to quantify its performance in this scenario.

In practice, the equations of Froehlich (2008) have been widely used as the base standard for parametric breach models. Froehlich (2016) updated these equations with additional dam failure data, providing a new (albeit similar) set of equations. Should the user wish to take into account erodibility, failure mode or dam type they may also utilise the Xu & Zhang (2009) equations, however these are more complex. Regardless of the user's particular requirements, it is generally recommended to implement several different parametric models when determining breach parameters, for the purposes of comparison. If selecting values to use in further analysis, the user should always use results from a single parametric model, rather than picking and choosing values from different models. This is because for any given model, the breaching parameters will all be related to one another.

Due to the large uncertainties, and therefore potential inaccuracies in results, these parametric methods as a whole are best suited for the rapid screening of dams, where the user may want to quickly and effectively identify potential hazards in a large database of dams, or in low risk scenarios, where uncertainties will have a minimal effect. They can often act as a good starting point for carrying out more detailed analyses. As stated by Wahl (2004), it is still the case in present day that engineering judgement should be used when interpreting any results from these methods.



Figure 8 - Flooding caused by the breaching of Testalinden dam. Image source: Paul Everest / Osoyoos Times (2010).

It is important to note that the underlying basis of these parametric models (historic failures of dams only) means they are not applicable to the breaching of levees. Fluvial and coastal levees, referring to those that border rivers and coastlines respectively, can however be modelled in several of the physically based models detailed below in this report. Fluvial levees will generally fail due to either internal erosion or overflow, with the flow of water parallel to the embankment. Coastal levees, on the other hand may fail due to internal erosion, overflow or overtopping failure, with the flow of water perpendicular and acting against the embankment.

5. Breach prediction methods - semi-physically based models

5.1. Overview

The purpose of these models is to add elements of a physical process to a breach simulation, whilst minimising the computational requirements (Morris, 2009). An outflow hydrograph will be generated with either erosion rates or breach geometry parameters as an input.

While this method may seem more accurate than using parametric models to directly determine peak outflow, and then estimating the outflow hydrograph, these results will only be as reliable as the data provided. By giving details of the erosion rate or the breach geometry growth / limits, the user is essentially defining the breach process. A semi-physically based model will simply calculate the flow that would occur through such a breach.

5.2. Models

5.2.1. HEC-RAS

The Hydrologic Engineering Centre's River Analysis System, developed by USACE (United States Army Corps of Engineers), is a common tool used for the study of rivers, flooding and dams (figure 9). It is primarily used for its one-dimensional and, more recently, two-dimensional flood routing methods. It is also a popular program for the use of dam break studies due to its ease of use and accessibility. These studies consider the breaching process of the dam semi-physically.

It is important to distinguish HEC-RAS from physically based models, such as EMBREA, WinDAM and DL Breach. These all function purely as a method of determining the characteristics of a breach, usually the outflow hydrograph and breach shape and progression, from a range of geometric, hydraulic and soil input parameters. HEC-RAS, on the other hand, is a flow routing model which contains a semi-physical breach model. Two options are available to a user in this model to estimate a breach hydrograph, described below in sections (a) and (b). These correspond to using breach geometry values and erosion rates as input parameters respectively.

1. **User Entered Data:** This option is the simplest and allows the user to enter the pre-determined breach geometry parameters. These will have been determined using one of the parametric methods detailed above in this report, or something similar. The user may also define the relationship between time and the breach progression (e.g. linear, curved etc.).
2. **Simplified Physical:** This option is more complex, but takes account of material properties somewhat. Users enter the maximum bounding breach width and height and do not define a breach formation time. They must then input relationships between the velocity of water and the down cutting and widening rates

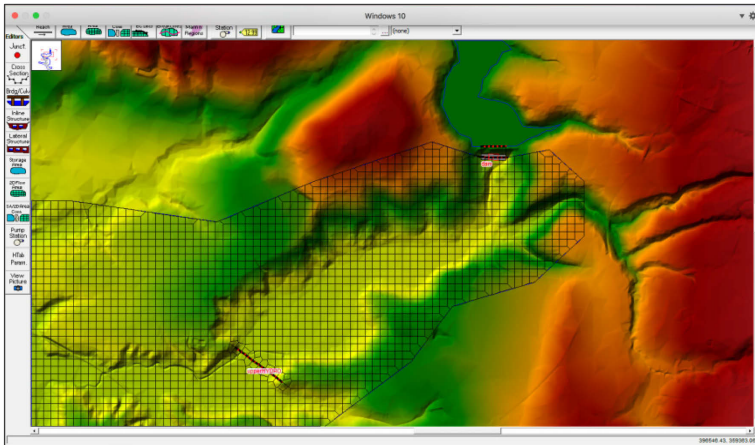


Figure 9 - Using HEC-RAS for semi-physical dam breach studies. Image shows the geometric data window, where a reservoir, dam and two-dimensional flow area have been defined over a digital terrain model.

6. Breach prediction methods - physically based models

6.1. Overview

Physically based models combine key hydraulic, structural and geotechnical properties, often in conjunction with empirical values and coefficients, to analytically or numerically predict the breaching process for an embankment. Many of these models have been developed in the last few decades.

The main overall advantage of these methods is the increased accuracy. Models such as EMBREA and DL Breach take many factors into account to closely predict the behavior and characteristics of an embankment dam breach, including erosion, sediment transport and slope stability. This produces breach hydrographs and geometry values with a greater certainty than those obtained in parametric and semi-physically based models.

As such, a great amount of time and investment has been put into solidifying and improving these models. Many laboratory (figure 10) and field studies, such as the IMPACT project (Morris et al, 2005) and USDA-ARS (Hanson et al, 2010, 2005a) research programme, have provided essential understanding of how dam breaches develop. HEC's guidance on performing dam break studies, as of 2014, recommends using physically based models where appropriate (specifically HR Breach (now EMBREA), WinDAM and NWS Breach) for determining breach characteristics, rather than parametric models (Brunner, 2014).

The working group on embankment dam erosion and breach modelling, organised under the Dam Safety Interest Group of CEATI International (which was intended to bring together a number of organisations with strong research programs in breach modelling, such as to advance the field) identified two physically based breach models as the most promising for future developments; HR Breach and SIMBA (now WinDAM) (DSIG, 2017).

There are several potential drawbacks that a user may want to consider to this breach modelling approach. Firstly, it should be recognised that the results of physically based models will reflect the reliability of the data

provided. A user should be careful and not assume that a physically based models results will be correct just because the underlying method is more rigorous. However, it should also be recognised that the added functionality and complexity of the model will also allow for the user to assess how different breach model parameters affect the breach prediction. Since some analyses will show less sensitivity to breach parameters than others (because of the site specific design, state and load conditions) this may allow the user to accept a greater uncertainty in some modelling parameters with a minimal effect on the breach predictions.

Whilst Zhong et al (2016) claimed that while an adequately developed and tested physically based model can outperform a parametric model, an underdeveloped and inaccurate physically based model will not always perform as well as a parametric model, particularly those of a more advanced nature, it is far from clear when or if such a claim can be validated.

Due to the significant uncertainties present in parametric breach prediction methods, this report would generally recommend using a physically based model with assumed values for unknown parameters over a parametric model. A sensitivity (or Monte Carlo) analysis should be carried out to assess the impact of the assumed values.



Figure 10 - Smaller laboratory experiments are often used in conjunction with larger field studies when validating breach models. Image source: Walder et al (2015).

Another factor that may make physically based models less attractive is the additional time required to obtain results. For example, users may opt for the simpler breach equations if, as part of a large study, they require the rapid assessment of many different embankment dams. Physically based models may also not be necessary when considering low risk situations, such as those with a low downstream flood risk, where uncertainty in breach predictions has a minimal effect on the flood modelling outcomes.

As mentioned in section 1, physically based models are often subdivided into two categories; those that are simple and those that are more complex. Simple models can be computed numerically or analytically, meanwhile detailed models are generally computed numerically only (Wu et al, 2011). Appendix C provides a list of all physically based models. This report will assess the usability of several physically based models that are available for public or commercial use.

6.2. Models

6.2.1. EMBREA

EMBREA is the primary embankment breaching program developed at HR Wallingford (Mohamed, 2002 & Hassan et al, 2002), UK. It can model overtopping failures for homogeneous, composite and layered embankments, and internal erosion failures for homogeneous and layered embankments. Embankments can be defined as cohesive or non-cohesive when considering erosion in the model. It also has the ability to model the failure of surface protection layers, such as grass cover, leading to dam breach. Monte Carlo Simulations may be computed to produce a probability distribution of results.

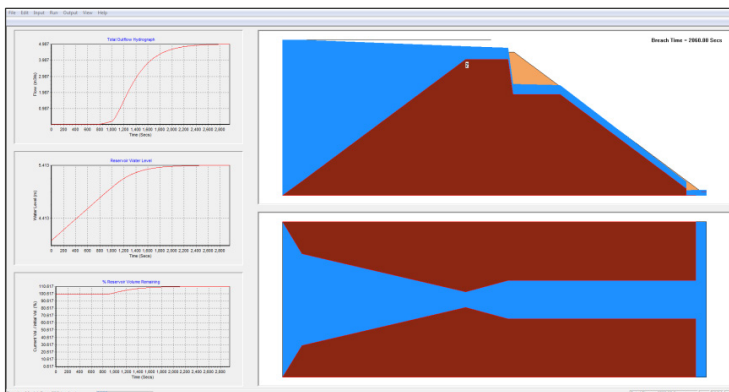


Figure 11 - EMBREA graphical user interface during head cut erosion simulation of an embankment.

EMBREA has several unique features which should be of interest to the reader. Unlike some other models detailed in this report, EMBREA is capable of modelling mass and micro failure of banks due to soil slope instability during the breach growth process. It also does not assume a predefined geometry (typically assumed as trapezoidal, triangular or rectangular in other simplified models) when predicting the breach growth. Multiple zones of material with differing erodibilities may be modelled, allowing more complex and modern dam constructions to be analysed without the need for simplifications.

Advantages:	Good balance of flexibility, speed and complexity of computations; can compute breaching of cohesive and non-cohesive embankments, with varying compositions and failure modes; Breach development is not predefined.
Disadvantages:	Simple one-dimensional flow calculations.
Usability:	Highly usable graphical user interface; Input parameter wizard makes entering data simple; Breach process is shown graphically as model computations progress; Extensive guidance on input parameters provided within program; Errors or instability issues are often described to the user, aiding model refinement; Allows the user to check the model before running; Results viewable in program.
Performance:	The model was validated against 4 historical cases, 7 field cases and 18 laboratory cases. The resulting performance was considered to be good. It was selected by DSIG (Dam Safety Interest Group) as one of the three most promising breach models for closer evaluation and overall conclusions were that the model, along with SIMBA (now incorporated into WinDAM) performed best and offered the greatest opportunity for future industry use (DSIG, 2017) (Morris, 2011).

6.2.2. AREBA

AREBA was developed under the FRMRC program (Van Damme et al, 2012). It is able to model overtopping failures in homogeneous and composite embankments and internal erosion failures in homogeneous embankments. For overtopping failures, surface erosion or head cut erosion can be modelled, as well as the failure of grass protection layers. Simple slope stability equations are also calculated.

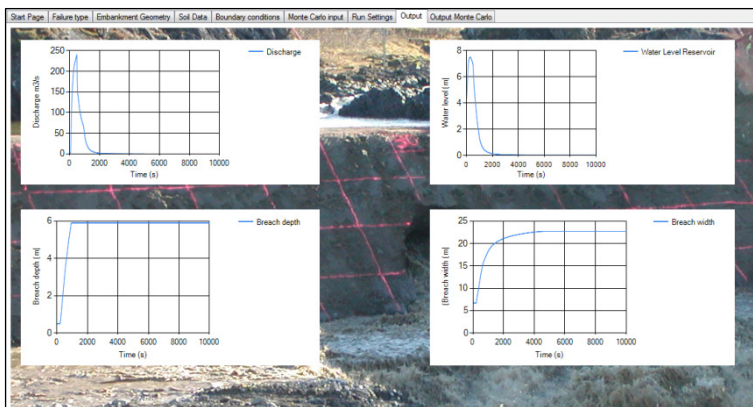


Figure 12 - AREBA graphical user interface displaying results from the overtopping breach analysis of a homogeneous, cohesive dam.

The primary advantages of AREBA are its simplicity, ease of use and speed. The model requires few inputs and has extremely quick run times. As such, while it may not be able to handle models that are as complex as those considered by other programs, it does allow the user to experiment and adapt models with greater ease. One disadvantage, in comparison with EMBREA, is that the breach development geometry is predefined.

Advantages:	Very fast run times; Computationally efficient; Few input parameters.
Disadvantages:	Fewer capabilities can mean some complex dam breaches must be simplified; Breach development must be predefined.
Usability:	Simple parameter input process and graphical user interface; Not overly complicated like some other models; Results viewable in program.
Performance:	The model was validated against a number of laboratory experiments and field case study data (IMPACT and USDA-ARS experiments), as well as the results of the EMBREA model.

6.2.3. DL Breach (simplified)

DL Breach is a dam and levee breach model developed by Wu (2013, 2016a). It is a relatively recent development in physical breach modelling and claims to have improved capabilities over its competition, including NWS Breach, HR Breach and WinDAM (however, open validation yet to be carried out).

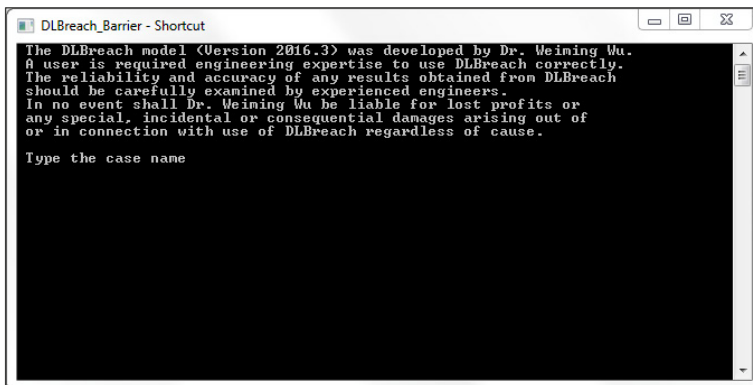


Figure 13 - Command prompt interface of DL Breach, where a user enters the name of the relevant input file.

The model is capable of simulating overtopping and internal erosion failures in homogeneous and composite earthen embankments. It may also model two-way breaches for coastal and levee breach purposes, where flows can reverse. As with EMBREA, the model can simulate failure of both cohesive and non-cohesive embankments, the failure of cover surfaces and can perform Monte Carlo simulations. A feature that may be of interest is the ability to model scour holes at the embankment breach. This was claimed by Zhong et al (2016) to be a significant factor.

Advantages:	Can simulate two-way breaches for levees and coastal defences; Ability to model base erosion (scour) of an embankment (usefulness not properly determined).
Disadvantages:	Breach development must be predefined.
Usability:	No graphical user interface, users must enter data in a text file and input this into the command prompt application; Information on how to format input file is available (Wu, 2016b); No information provided on errors or instabilities.
Performance:	Wu (2013) validated the DL Breach model against 50 test cases, formed of laboratory and field case study data. It was claimed that 98, 80 and 69% of predicted values had an error of 25% or less, for peak outflow, breach width and time to failure respectively.

6.2.4. WinDAM C

WinDAM C is developed by the Agricultural Research Service and the Natural Resources Conservation Service in the U.S. WinDAM has the capability to model overtopping and internal erosion in embankment dams, head cut erosion and the failure of protection layers. The program has a unique feature in that it can model an embankment with multiple spillways. A major limitation of WinDAM C is that it is currently limited to homogeneous dams only.

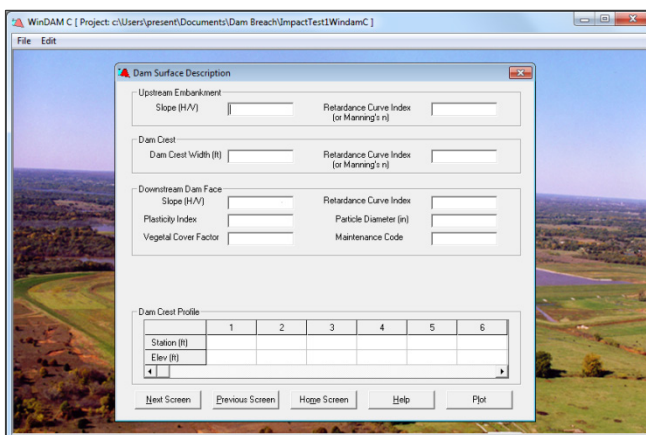


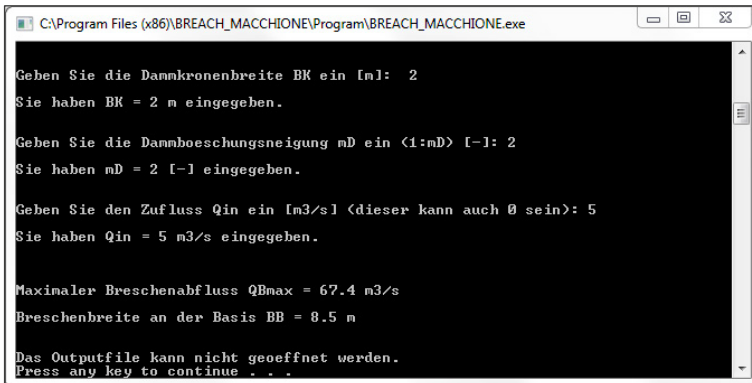
Figure 14 - WinDAM graphical user interface during the parameter input stage.

Future plans for WinDAM versions involve adding modelling capabilities for embankment slope protection materials (concrete blocks, reinforced vegetation etc.) and zoned fill embankment overtopping erosion prediction (Visser, 2013).

Advantages:	Capability to model erosion in multiple earthen auxiliary spillways.
Disadvantages:	Can only model homogeneous earthen dams; Breach development must be predefined.
Usability:	Allows the user to check the model before running; Most data can be entered only in imperial units; Otherwise very simple parameter input process; Guidance available in program for all input parameters.
Performance:	The model has been, and continues to be rigorously validated against a number of experimental and field test cases, notably those of the USDA-ARS (U.S. Department of Agriculture – Agricultural Research Service).

6.2.5. Macchione Breach

Macchione Breach was developed at ETH Zurich (Swiss Federal Institute of Technology) and can estimate breach parameters and peak outflow for the progressive breaching of homogeneous embankment dams by overtopping and internal erosion.



```

C:\Program Files (x86)\BREACH_MACCHIONE\Program\BREACH_MACCHIONE.exe
Geben Sie die Dammkronenbreite BK ein [m]: 2
Sie haben BK = 2 m eingegeben.

Geben Sie die Dammböschungsnäigung mD ein <1:mD> [-]: 2
Sie haben mD = 2 [-] eingegeben.

Geben Sie den Zufluss Qin ein [m3/s] <dieser kann auch 0 sein>: 5
Sie haben Qin = 5 m3/s eingegeben.

Maximaler Breschenabfluss QBmax = 67.4 m3/s
Breschenbreite an der Basis BB = 8.5 m

Das Outputfile kann nicht geöffnet werden.
Press any key to continue . . .
  
```

Figure 15 - Command prompt interface of Macchione Breach, showing resulting output values for peak outflow and bottom width of the breach.

This model is reportedly currently being incorporated into the software tool BASEbreach, which will replace the Macchione Breach application in the near future.

Advantages:	Few input parameters
Disadvantages:	Model requires calibration parameter; Breach development must be predefined; Does not model slope stability.
Usability:	Program and accompanying manuals available in German language only; No graphical user interface, similarly to DL Breach. Accompanying manual provides details on how to format input files.
Performance:	The model was validated against 12 historic dam failure cases.

6.2.6. BASEMENT / Volz et al (2012)

BASEMENT (Basic Simulation Environment) is a river and sediment flow simulation tool available through ETH Zurich as freeware. Volz et al (2012) created an embankment breach model using the underlying code of BASEMENT. The model uses two-dimensional flow and surface erosion calculations, as well as three-dimensional slope stability equations.

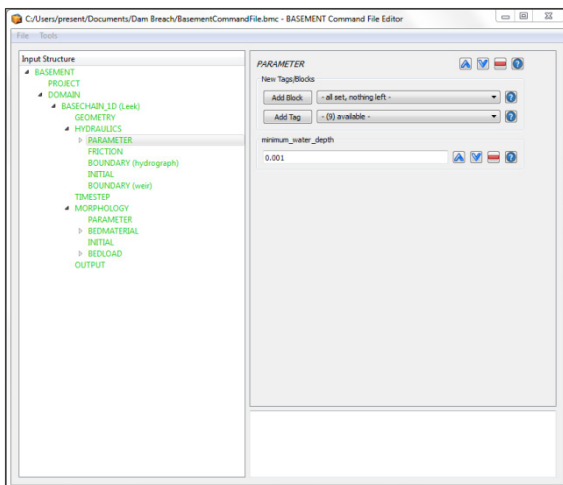


Figure 16 - BASEMENT model graphical user interface during the parameter input stage.

While not publicly available at the time of writing, it is anticipated that this breach model may be officially incorporated into BASEMENT in the future.

Advantages:	Models seepage and surface flow; Two-dimensional flow and erosion calculations; Three-dimensional slope stability.
Disadvantages:	Needs very low time-step, therefore computationally expensive; Limited to non-cohesive embankments.
Usability:	Structured, step-by-step input process in BASEMENT; Guidance on input parameters provided in program.
Performance:	The model was validated against two laboratory experiments and a field study breach (IMPACT project).

6.3. Existing comparisons of physically based models

Existing technical papers can provide an idea of the relative performance of physically based breach models in comparison to each other. However, these comparisons are often carried out as part of work representing new research and new physically based models, and frequently present only part of the overall picture. Results are often based on the use of older (more easily available) breach models or the inappropriate application and use of more recent models. It is therefore important to consider this when interpreting any results and ideally to look for independent validation of breach model performance where possible.

Zhong et al (2016) [co-authored with the creator of DL Breach] compared the performance of NWS Breach (1988), HR Breach (version 4.1, 2008) and DL Breach (2016) in 12 test cases, which were compiled from historical dam failures and several field tests.

From the analysis carried out, it was claimed by Zhong et al that DL Breach provided the most accurate results and NWS Breach provided the least accurate results, with HR Breach being in the middle in terms of performance. This result might be expected, as newer physically based models will often outperform those that are many years older (at the time of writing, the version of HR Breach compared in this study is outdated by almost a decade, and the version of NWS breach by almost three decades).

It was also found that in the same study that soil erodibility was one of the most important parameters when physically modelling dam breaches. Each model was sensitive to this value, with HR Breach and DL Breach being more so than NWS Breach. As such, regardless of model used, it is important that the uncertainty in soil erodibility should be reduced where possible.

Zhong et al (2017) proposed a simplified model for homogeneous cohesive embankment breaching due to overtopping failure. The newly proposed model was evaluated, alongside HR Beach (version not specified, 2012 or earlier), WinDAM B (2011) and NWS Breach (1988), against the data of two field studies and one historic failure case. The results of this comparison are given below in table 4.

The study claimed the newly proposed model to perform the best, NWS Breach the worst, and WinDAM B and HR Breach between the aforementioned two in terms of accuracy. It was concluded that, owing to significant uncertainties in the measured data, performances of models will vary case by case, meaning the selection of an embankment breach model will depend on the capabilities of the model and the characteristics of the scenario to which it is applied.

It is also worth noting that this model was developed and calibrated with the same NHRI field test data that is sampled in Table 4 below, which may explain (or partly explain) the accuracy of the result. Additionally, as with Zhong et al (2016), the reader should recognise that the newly proposed model has been compared against various other older models for which newer model versions are available. A better performance comparison would have been achieved by using current versions for all of the models.

Table 4: Performance comparison between Zhong et al, HR Breach, WinDAM B and NWS Breach, adapted from Zhong et al. (2017).

Breach Model	NHRI Field #2			USDA-ARS #1			Goose Creek, USA		
	Q_p (m^3/s)	B_{top} (m)	t_p (hr)	Q_p (m^3/s)	B_{top} (m)	t_p (hr)	Q_p (m^3/s)	B_{top} (m)	t_p (hr)
Recorded	42.3	17.0	0.20	6.5	6.9	0.67	565.0	30.5	-
Zhong et al.	41.5	21.5	0.30	5.5	8.2	0.65	623.7	44.4	2.85
HR Breach	274.6	26.2	0.15	4.7	8.2	0.83	669.7	27.9	2.84
WinDAM B	171.5	18.8	0.21	6.0	6.4	0.55	481.0	31.7	3.74
NWS Breach	423.3	18.0	1.03	16.1	3.0	0.27	259.8	24.0	1.01



Figure 17 - Field studies carried out as part of the IMPACT Project (Morris et al 2005) were used to develop the HR Breach model. Image source: Morris (2011)



Figure 18 - Zhong et al (2017) validated their model using results from NHRI (Nanjing Hydraulic Research Institute) dam breach field tests. Image source: Zhang et al (2009).

7. How to choose and apply a breach prediction method

7.1. Overview

Choosing a breach prediction method is ultimately a case of balancing risk and uncertainty and the different methods that will be best suited for different applications. Guess work and rule of thumb relationships can be used when breaches similar to the scenario in question have occurred in the past, but are generally not applicable or appropriate to breach modelling.

Parametric methods predict breach characteristics with a high uncertainty and so should generally not be used for high risk scenarios. Their simplicity and ease of application does, however, lend them for rapid, low risk flood assessment.

Semi-physically based models will reflect the uncertainty of the breach geometry or erosion values they are provided, as they simply use these values to generate a breach outflow hydrograph. As such, these are typically also suitable for low risk flood assessment. For both parametric and semi-physical models, a principle of 'worst case scenario' is often used to help negate the uncertainties present.

Physically based models have a greatly reduced uncertainty over parametric and semi-physical models, due to the complex underlying mathematical processes. These should therefore be used for higher risk scenarios, or those where accurate and reliable results are required.

Appendix A gives a recommended approach to choosing a breach prediction method.

7.2. Examples

Design and construction of a Nuclear Power Station downstream of a large reservoir

The risks associated with dam breach and consequent flooding in this scenario are extremely high, therefore the uncertainty required from the prediction method should be low, and a physically based method should be used.

Flood risk assessment for levee breach along a countryside river

The impact of breach flooding will be dependent on the height of water impounded by the levee and any infrastructure or buildings in the immediate area of the river, however it will likely be low. A parametric method or data taken from similar past levee breaches coupled with rules of thumb would therefore be suitable.

Emergency planning for a housing development 10 km downstream of a medium-sized reservoir

While the flood risk may still be high, dependant on the positioning of the housing development with respect to the natural river path, the large distance will make the sensitivity of this area to different breach outflow hydrographs negligible. As such, it would be appropriate to utilise a semi-physically based method.

Emergency planning for a housing development immediately downstream of a medium-sized reservoir

In this scenario, the sensitivity to the breaching process will be extremely high, therefore a minimum level of uncertainty is essential to accurately model the impact of flooding on this area. A physically based method should therefore be most appropriate.

System risk modelling of a large inventory of dams

In cases where a large number of studies must be carried out, parametric models are typically the most ideal initial course of action for predicting breach characteristics, due to their speed and efficiency. However, the physical model AREBA was designed specifically with system risk modelling in mind, and its fast run times would make it the ideal choice.

8. Conclusions

This paper has provided an overview into the different methods available to a user when modelling the breaching of earth embankments. The strengths and weaknesses of the various breach prediction methods available have been described, such that the appropriate model may be chosen for a given scenario.

Parametric models are simple, easy to use equations which can be used to determine the potential size of a breach and its peak outflow. These can be estimated using simple reservoir and dam geometry data, which is often easier to obtain than the more complex soil data required for physically based models.

When compared to several relatively recent embankment breaches, the performance of the various equation sets was varied. Froehlich (1995a, b), (2008) and Von Thun & Gillette (1990) provided reasonable results that can be considered within an acceptable tolerance (for such equations), meanwhile the more basic USBR (1982, 1988) equations fell short of providing accurate values. Xu & Zhang's (2009) model is one of the few to take soil erodibility and several other dam and material factors into account, which is claimed to improve reliability. This was indeed found by Zhong et al (2016), however the values estimated using this method in table 3 of this report were not as accurate as others. In all cases, large errors and uncertainties still mean that these methods should be treated with caution, and engineering judgement must be exercised when interpreting their results (Wahl, 2004).

Physically based models, while slower and more labour-intensive, have the ability to predict complex dam breach scenarios in a more accurate manner. Several of the more prominent and sophisticated breach models were considered and compared in this report, providing individuals with recommendations and guidance on the applicability and suitability of each method to various dam breach scenarios. More rigorous results tend to be producible with these types of methods, when compared to parametric models, such as full breach hydrographs and plots of breach size evolution against time.

The most suitable physically based model will depend on the application. AREBA's simplicity and usability, whilst still maintaining a rigorous and complex computation process, makes it ideal for non-complex embankment breach scenarios, such as homogeneous and non-cohesive core-wall dams. EMBREA is more complex and requires a greater number of input parameters, but has the functionality to competently model a much larger variety of breaches, such as those in dams comprising of multiple layers of material with differing erodibilities. It is also one of the few models to not predefine breach geometry development; rather it relates it to the effective shear stress of the dam material. Overall, a more in-depth analysis can be carried out, and fewer simplifications are made than in AREBA, providing results with a greater accuracy and a method that is a desirable balance of capability and usability.

DL Breach (simplified) has good functionality and complexity and can be considered flexible due to the wide range of input parameters that can be used. It is also the only model to consider base erosion (scour) at the embankment breach. However, it does lack usability and can be time consuming when encountering model instability and errors, as no information on these issues is provided to the user.

Although limited in its applicability, WinDAM is a highly usable and stable model. At the time of writing, the model is not recommended unless considering head cut erosion in simple, homogeneous earthen embankments. Macchione Breach, in its current version, is also not recommended due to the lack of usability and applicability.

There is a great deal of room for further technological developments in physical dam breach modelling, which is expected to improve the accuracy and capabilities of the models described in this report. More complex models, such as that proposed by Volz et al (2012), perform multi-dimensional calculations when

considering the movement of flow and sediment and employ sophisticated slope stability and seepage equations. The anticipated integration of physical breach modelling with flood routing software is also likely to improve the usability of the dam breach flood modelling process as a whole.

Regardless of which method a user adopts, uncertainties will be present. Great progress has been made recently through technological advances and several relatively large-scale field studies, such as the IMPACT project and the USDA-ARS research programme. These have improved the capabilities of physically-based models, where understanding of erosion and sediment transport behaviour in dam breaches has often been lacking. More large scale physical tests should be carried out to progressively improve both simple and more complex models in the future, reducing their uncertainty and improving their effectiveness at predicting embankment breaches.

9. Notation

\bar{B}	=	Average width of final breach (m)
B_{top}	=	Top width of final breach (m)
b_4	=	Variable coefficient dependant on failure mode (Xu & Zhang, 2009)
b_5	=	Variable coefficient dependant on erodibility (Xu & Zhang, 2009)
C_b	=	Offset coefficient, a function of reservoir volume (m) (Von Thun & Gillette, 1990)
e	=	Euler's constant, approximately 2.72
g	=	Acceleration due to gravity, approximately 9.81 m/s^2
h_b	=	Height of final breach (m)
h_d	=	Original height of dam (m)
h_r	=	15m (Considered as reference height to distinguish between small and large dams)
h_w	=	Height of water above final breach bottom, at time of failure (m)
k_0	=	Variable coefficient dependant on failure mode (Froehlich, 1995a, 2008, 2016a, b)
k_H	=	Embankment height factor (Froehlich, 2016b)
Q_p	=	Peak outflow from breach (m^3/s)
t_f	=	Breach formation time (hr)
t_p	=	Time to peak breach outflow (hr)
t_r	=	1 hour (unit duration)
V_{er}	=	Volume of embankment material eroded (m^3)
V_r	=	Total volume of water contained in reservoir (m^3)
V_w	=	Volume of water contained in reservoir above final breach bottom, at time of failure (m^3)
\bar{W}	=	Average embankment width (m)
z	=	Side slopes of breach ($h:v$)
OT	=	Overtopping failure mode
P	=	Internal erosion / other failure mode
HE	=	High erodibility material
ME	=	Medium erodibility material
LE	=	Low erodibility material

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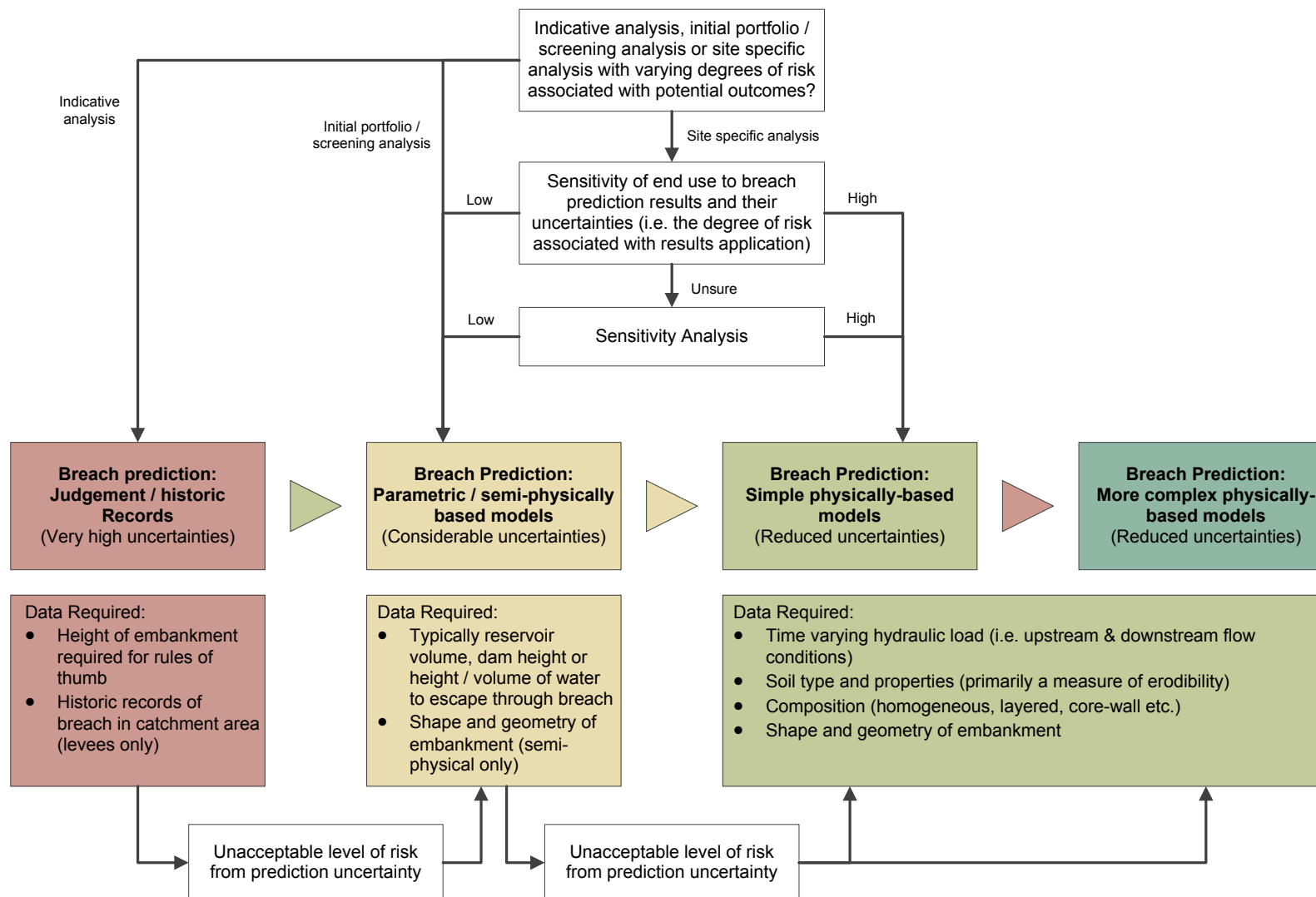
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Appendix A - Recommended approach for choosing a breach prediction method



Appendix B - Summary of parametric breach prediction models

Parametric Model	Time to Failure, t_f (hr)	Average breach width, \bar{B} (m)	Side Slopes, z (h: v)	Peak Outflow, Q_p (m^3/s)	Number of Case Studies
<i>Kirkpatrick (1977)</i>				$Q_p = 1.268(h_w + 0.3)^{2.5}$	16 (plus 5 hypothetical)
<i>Soil Conservation Service (1981)</i>				$Q_p = 16.6h_w^{1.85}$	13
<i>Hagen (1982)</i>				$Q_p = 0.54(V_r h_d)^{0.5}$	6
<i>U.S. Bureau of Reclamation (1982, 1988)</i>	$t_f = 0.011\bar{B}$	$\bar{B} = 3h_w$		$Q_p = 19.1h_w^{1.85}$	21
<i>Singh and Snorrason (1984)</i>				$Q_p = 13.4h_d^{1.89}$ $Q_p = 1.776V_r^{0.47}$	28 (20 real, 8 simulated)
<i>MacDonald and Langridge-Monopolis (1984)</i>	$t_f = 0.0179V_{er}^{0.364}$	<p>For earth fill dams: $V_{er} = 0.0261(V_w h_w)^{0.769}$</p> <p>For rock fill dams: $V_{er} = 0.00348(V_w h_w)^{0.852}$</p>		$Q_p = 1.154(V_w h_w)^{0.412}$	42
<i>Costa (1985)</i>				$Q_p = 0.981(V_r h_d)^{0.42}$	31
<i>Evans (1986)</i>				$Q_p = 0.72V_w^{0.53}$	
<i>Von Thun & Gillette (1990)</i>	<p>Highly erodible: $t_f = 0.015h_w$ $t_f = \bar{B}/(4h_w + 61)$</p> <p>Erosion resistant: $t_f = 0.015h_w + 0.25$ $t_f = \bar{B}/(4h_w)$</p>	<p>$\bar{B} = 2.5h_w + C_b$</p> $C_b = \begin{cases} 6.1 & (V_r \times 10^{-6}) < 1.23 \\ 18.3 & 1.23 \leq (V_r \times 10^{-6}) < 6.17 \\ 42.7 & 6.17 \leq (V_r \times 10^{-6}) < 12.3 \\ 54.9 & 12.3 \leq (V_r \times 10^{-6}) \end{cases}$			57

Parametric Model	Time to Failure, t_f (hr)	Average breach width, \bar{B} (m)	Side Slopes, z (h: v)	Peak Outflow, Q_p (m^3/s)	Number of Case Studies
Froehlich (1995a, 1995b)	$t_f = 0.00254V_w^{0.53}h_b^{-0.9}$	$\bar{B} = 0.1803k_0V_w^{0.32}h_b^{0.19}$ $k_0 = \begin{cases} 1.4 & OT \\ 1.0 & P \end{cases}$	$z = \begin{cases} 1.4 & OT \\ 0.9 & P \end{cases}$	$Q_p = 0.607V_w^{0.295}h_w^{1.24}$	1995a: 22, 1995b: 63
Walder & O'Connor (1997)				$Q_p = a(h_wV_w)^b$ where: $a, b = \begin{cases} 0.99, 0.40 & \text{Landslide} \\ 0.61, 0.43 & \text{Constructed} \\ 0.19, 0.47 & \text{Moraine} \end{cases}$	
Froehlich (2008)	$t_f = 0.0176\sqrt{\frac{V_w}{gh_b^2}}$	$\bar{B} = 0.27k_0V_w^{\frac{1}{3}}$ Where: $k_0 = \begin{cases} 1.3 & OT \\ 1.0 & P \end{cases}$	$z = \begin{cases} 1.0 & OT \\ 0.7 & P \end{cases}$		74
Xu & Zhang (2009)	$\frac{t_f}{t_r} = C_5 \left(\frac{h_d}{h_r}\right)^{0.654} \left(\frac{V_w^{1/3}}{h_w}\right)^{1.246}$ where: $C_5 = b_5$ $b_5 = \begin{cases} 0.038 & HE \\ 0.066 & ME \\ 0.205 & LE \end{cases}$	$\frac{\bar{B}}{h_b} = 5.543 \left(\frac{V_w^{1/3}}{h_w}\right)^{0.739} e^{C_3}$ where: $C_3 = b_4 + b_5$ $b_4 = \begin{cases} -1.207 & OT \\ -1.747 & P \end{cases}$ $b_5 = \begin{cases} -0.613 & HE \\ -1.073 & ME \\ -1.268 & LE \end{cases}$	1	$\frac{Q_p}{\sqrt{gV_w^{5/3}}} = 0.133 \left(\frac{V_w^{1/3}}{h_w}\right)^{-1.276} e^{C_4}$ where: $C_4 = b_4 + b_5$ $b_4 = \begin{cases} -0.788 & OT \\ -1.232 & P \end{cases}$ $b_5 = \begin{cases} -0.089 & HE \\ -0.498 & ME \\ -1.433 & LE \end{cases}$	75
Pierce at al. (2010)				$Q_p = 0.0176(V_w h_w)^{0.606}$ $Q_p = 0.038(V_w^{0.475} h_w^{1.09})$	87
Froehlich (2016a, b)	$t_f = 60\sqrt{\frac{V_w}{gh_b^2}}$	$\bar{B} = 0.23k_0V_w^{\frac{1}{3}}$ Where:	$z = \begin{cases} 1.0 & OT \\ 0.6 & P \end{cases}$	$Q_p = 0.0175k_0k_H\sqrt{\frac{gV_w h_w h_b^2}{W}}$ Where:	2016a: 111, 2016b: 41

Parametric Model	Time to Failure, t_f (hr)	Average breach width, \bar{B} (m)	Side Slopes, z (h: v)	Peak Outflow, Q_p (m^3/s)	Number of Case Studies
		$k_0 = \begin{cases} 1.5 & OT \\ 1.0 & P \end{cases}$		$k_0 = \begin{cases} 1.85 & OT \\ 1.0 & P \end{cases}$	
				$k_H = \begin{cases} 1 & h_b < 6.1 \\ (h_b/6.1)^{1/8} & h_b > 6.1 \end{cases}$	

Appendix C - Summary of physically based embankment breach prediction model

Method	Breach Morphology	Flow over Dam	Sediment Transport	Geomechanics of breach side-slopes	Limitations	Availability	Main Publication(s) / References
<i>Cristofano (1965)</i>	YZ: Trapezoidal, constant bottom width XZ: Constant downstream slope	Weir formula	Cristofano's formula	None	<ul style="list-style-type: none"> Constant breach bottom width and shape No lateral erosion or slope stability analysis Unrealistic erosion relation 	?	Cristofano (1965)
<i>Harris and Wagner (1967)</i>	YZ: Parabolic with top width = 3.75 x depth XZ: Constant downstream slope	Weir formula	Schoklitsch formula	None	<ul style="list-style-type: none"> Constant sedimentation concentration No slope stability analysis User input breach slope 	?	Harris and Wagner (1967)
<i>BRDAM</i>	YZ: Parabolic XZ: Constant downstream slope	Weir formula for overtopping and orifice for IE.	Schoklitsch's formula	Top wedge failure during IE, no lateral collapse	<ul style="list-style-type: none"> No lateral erosion or slope stability analysis Constant sedimentation concentration User input time for IE failure 	?	Brown and Rodgers (1981); developed from Harris and Wagner (1967)
<i>Ponce and Tsivoglou (1981)</i>	YZ: function of flow rate XZ: Exner equation	Unsteady St Venant equations	Meyer-Peter- Mueller formula	None	<ul style="list-style-type: none"> No slope stability analysis No lateral erosion after peak flow occurs 	?	Ponce and Tsivoglou (1981)
<i>Lou (1981)</i>	YZ: Cosine shape XZ: Exner equation	Unsteady St Venant equations	DuBoy's and Einstein's formulas	None	<ul style="list-style-type: none"> No slope stability analysis No lateral erosion after peak flow occurs Empirical formula used to compute erosion Breach growth not appropriately modelled 	?	Lou (1981)
<i>Nogueira (1984)</i>	YZ: Cosine shape XZ: Exner equation	Unsteady St Venant equations	Meyer-Peter- Mueller formula	None	<ul style="list-style-type: none"> Similar to Lou (1981), hence same limitations 	?	Nogueira (1984)
<i>DAMBRK</i>	YZ: Trapezoidal or rectangular XZ: Constant downstream slope	Weir formula	Assumed linear erosion	None		?	Fread (1984)
<i>SMPDBK</i>	YZ: Rectangular	Weir formula	None	None		?	Wetmore and Fread (1984)

Method	Breach Morphology	Flow over Dam	Sediment Transport	Geomechanics of breach side-slopes	Limitations	Availability	Main Publication(s) / References
<i>BEED</i>	YZ: Trapezoidal XZ: Constant downstream slope and erosion of the crest	Weir formula	Einstein and Brown, Meyer-Peter-Mueller formulas	Breach side slope stability	<ul style="list-style-type: none"> Uniform erosion of breach Incompatible computation method for hydraulics and sediment Innacurte slope stability analysis 	?	Singe and Scarlatos (1985)
<i>EMBANK</i>	Erosion in horizontal layers, breach width undetermined	Weir equation, velocity profile	DuBoy's formula, Shields diagram	Lateral collapse effects	<ul style="list-style-type: none"> No mass conservation 	?	Chen and Anderson (1986)
<i>Fujita and Tamura (1987)</i>	Rectangular above water level and trapezoidal below	Weir formula	Estimating assuming energy slope consumed only in sediment transport	None	<ul style="list-style-type: none"> Uniform erosion of breach 	?	Fujita and Tamura (1987)
<i>Singh and Scarlatos (1988)</i>	YZ: Trapezoidal, rectangular or triangular	Weir formula	Erosion rate as a function of flow velocity			?	Singh and Scarlatos (1988)
<i>NWS Breach</i>	YZ: Trapezoidal or rectangular XZ: Constant downstream slope	Weir formula for overtopping and orifice for IE	Meyer-Peter-Mueller, modified by Smart	Breach side slope stability, top wedge failure during IE	<ul style="list-style-type: none"> Predefined breach development Simplified modelling for failure of composite embankments Innaccurate slope sability analysis Uniform erosion of breach Incompatible computation method for hydraulics and sediment 	Free	Fread (1988)
<i>Giuseppetti and Molinaro (1989)</i>	Triangular	Weir formula	Engelund and Smart			?	Giuseppetti and Molinaro (1989)
<i>Havnø et al. (1989)</i>	Trapezoidal		Linear predetermined Engelund and Hanson, Meyer-Peter-Mueller			?	Havnø et al. (1989)
<i>Tingsanchali and Hoai (1993)</i>		Weir formula	Meyer-Peter-Mueller, Exner equation	None			Tingsanchali and Hoai (1993)
<i>SITES</i>	XZ: Three stages 1. Cover failure 2. Headcut formation 3. Headcut erosion	Spillway stage-discharge curve	Stage 1-2: Detachment model Stage 3: Energy dissipation equation	Spillway exit channel stability	<ul style="list-style-type: none"> Incomplete modelling of embankment failure Erosion dependant on emperical coefficient 	?	NRCS (1997)
<i>DEICH_A</i>	YZ: Trapezoidal XZ: Horizontal channel	Weir formula	Meyer-Peter-Mueller	None	<ul style="list-style-type: none"> Non predictive capability 	?	Broich (1998)
<i>ED Breach</i>	YZ: Trapezoidal	Weir flow for overtopping,	Meyer-Peter-Mueller	Top wedge failure during IE		?	Loukola and Huokuna (1998)

Method	Breach Morphology	Flow over Dam orifice for IE	Sediment Transport	Geomechanics of breach side-slopes	Limitations	Availability	Main Publication(s) / References
<i>DEICH_N1</i>	XZ: Exner equation	1D shallow water equations	Multiple formulas	None	<ul style="list-style-type: none"> Breach shape predefined No slope stability analysis Unrealistic modelling of erosion 	?	Broich (1998)
<i>DEICH_N2</i>	YZ: Diffusivity approach XZ: Exner equation	2D shallow water equations	Multiple formulas	None	<ul style="list-style-type: none"> Breach shape predefined No slope stability analysis Unrealistic erosion process 	?	Broich (1998)
<i>NCP-Breach</i>	YZ: Parabolic XZ: Constant downstream slope and erosion around pivot point	Weir formula	Empirical formula	None		?	Coleman and Andrews (1998), Coleman et al. (2002)
<i>RUPRO</i>	YZ: Rectangular XZ: Horizontal channel	Bernoulli equation	Meyer-Peter-Mueller	None		?	Paquier (1998), Paquier et al. (1998), Paquier (2002)
<i>BRES</i>	YZ: Trapezoidal XZ: Exner equation with rotation up to a constant downstream slope	Weir formula & Bélanger	Various equations for cohesive and non-cohesive soils	Simple slope stability	<ul style="list-style-type: none"> Slope stability calculations are simplified No effect of waves taken into account 	?	Visser (1998), Zhu et al. (2006)
<i>Peviani (1999)</i>	YZ: Trapezoidal XZ: Exner equation	1D shallow water equations	Di Silvio and Peviani	Slope stability model		?	Peviani (1999)
<i>BSTEM (Bank Stability and Toe Erosion Model)</i>					<ul style="list-style-type: none"> Not primarily a breach model 	Free	Simon et al. (2000)
<i>Riha and Danacek (2000)</i>	YZ: Rectangular	Analytical solution for breach outflow and reservoir water level	Ponce's approach	None		?	Riha and Danacek (2000)
<i>Tingsanchali and Chinnarasri</i>	XZ: Exner equation	1D shallow water equations	Multiple formulas	Longitudinal slope stability		?	Tingsanchali and Chinnarasri (2001)

Method	Breach Morphology	Flow over Dam	Sediment Transport	Geomechanics of breach side-slopes	Limitations	Availability	Main Publication(s) / References
<i>(2001)</i>							
<i>ERODE</i>	No assumptions	Stream tube formulisation	Minimum stream power – sediment transport formula			?	Marche and Fuamba (2002)
<i>HR Breach</i>	YZ: Effective shear stress dependant XZ: Exner equation & soil wasting	Variable weir formula & 1D Steady non-uniform flow equations	Various equations for cohesive and non-cohesive soils	Slope stability, core stability and multiple zones of variable erodibility	<ul style="list-style-type: none"> Does not model scour holes or seepage flow 	Commercial	Hassan (2002), Hassan et al. (2002)
<i>FIREBIRD</i>	YZ: Variable trapezoidal XZ: Exner equation	Unsteady St Venant equations	Sediment transport formulas or erosion rate equations	Slope stability	<ul style="list-style-type: none"> Limited model validation 	?	Wang and Kahawita (2002), Wang et al. (2006)
<i>Rozov (2003)</i>	YZ: Rectangular XZ: Exner equation	Weir formula	Sediment transport as function of flow velocity and depth	None		?	Rozov (2003)
<i>DaveF</i>	2D Exner equation	2D shallow water equations	Erosion formula from WEPP, USDA	None		?	Froehlich (2004)
<i>RoDaB</i>		Weir formula	Sediment transport as function of flow velocity			?	Franca and Almeida (2004)
<i>Kraus and Hayashi (2005)</i>	YZ: Rectangular XZ: Horizontal channel	1D Keulegan equation	Empirical formula and longshore sediment source	None		?	Kraus and Hayashi (2005)
<i>WinDAM / SIMBA</i>	YZ: Headcut development and migration XZ: Rectangular or trapezoidal	Weir formula	Parametric relations for headcut advance, bottom and lateral erosion	Breach side slope erosion	<ul style="list-style-type: none"> Predefined breach development Does not model seepage flow or slope stability Model needs soil erodability input value Can only model overtopping 	Free	Temple et al. (2005), Hanson et al. (2005b), Temple et al. (2006)
<i>Wang and Bowles (2006a, b)</i>		2D shallow water equations	Chen and Andreson's erosion rate formula	3D slope stability	<ul style="list-style-type: none"> Does not model seepage flow 	?	Wang and Bowles (2006a, b)

Method	Breach Morphology	Flow over Dam	Sediment Transport	Geomechanics of breach side-slopes	Limitations	Availability	Main Publication(s) / References
<i>D'Eliso (2007)</i>	YZ: Headcut development and migration XZ: Rectangular to trapezoidal	Wave overtopping and/or overflow – Bernoulli equation.	Formulas for erosion rate and headcut advance	Grass cover, clay cover, sand core and breach slope stability		?	D'Eliso (2007)
<i>DL Breach 1D</i>	1D Non-equilibrium total-load transport equation	Generalized shallow water equations	Wu et al. total-load capacity formula	Lateral erosion and slope stability (repose angle)	<ul style="list-style-type: none"> Does not model seepage flow or slope stability Limited model validation Focuses more on flood routing rather than breach development 	?	Wu and Wang (2007)
<i>Faeh (2007)</i>	2D Exner equation	2D shallow water equations	Formula for bed-load and suspended load	Lateral erosion, vertical erosion and slope stability		?	Faeh (2007)
<i>Macchione Breach</i>	YZ: Trapezoidal or triangular	Weir formula	Sediment transport as function of bed shear stress	None	<ul style="list-style-type: none"> Predefined breach development Does not model seepage flow or slope stability Models needs a calibration parameter 	Free	Macchione (2008) , Macchione and Rino (2008)
<i>Wang et al. (2008)</i>	2D Non-equilibrium sediment transport equation	2D shallow water equations	Formula for bed-load	Lateral erosion, vertical erosion and slope stability		?	Wang et al. (2008)
<i>Reelvink et al. (2009)</i>	2D Non-equilibrium sediment transport equation	2D shallow water equations with wave-action	Soulsby formula	Bed avalanching		?	Reelvink et al. (2009)
<i>MIKE-11</i>	YZ: Trapezoidal XZ: Exner equation	Weir flow for overtopping, orifice for IE	Engelund-Hansen			Commercial	Danish Hydraulic Institute (2009)
<i>Pontillo et al. (2010)</i>				None		?	Pontillo et al. (2010)
<i>DL Breach 2D</i>	2D Non-equilibrium total-load transport equation	Generalized shallow water equations		Lateral erosion and slope stability (repose angle)	<ul style="list-style-type: none"> Does not model seepage flow Limited model validation 	?	Wu (2010), Wu et al. (2012)

Method	Breach Morphology	Flow over Dam	Sediment Transport	Geomechanics of breach side-slopes	Limitations	Availability	Main Publication(s) / References
<i>Cao et al. (2011)</i>	2D Non-equilibrium sediment transport equation	Generalized shallow water equations	Modified Meyer-Peter and Mueller bed-load	Slope stability (repose angle)		?	Cao et al. (2011a, b)
<i>EMBREA</i>	YZ: Effective shear stress dependant XZ: Exner equation & soil wasting	Variable weir formula & 1D Steady non-uniform flow equations	Various equations for cohesive and non-cohesive soils	Slope stability, core stability and multiple zones of variable erodibility	<ul style="list-style-type: none"> Does not model scour holes or seepage flow 	Commercial	Morris, M.W (2011)
<i>AREBA</i>		Weir formula	Various equations for cohesive and non-cohesive soils	Simple slope stability	<ul style="list-style-type: none"> Predefined breach development Does not model seepage flow 	Commercial	Van Damme, Morris, Hassan (2012)
<i>BASEMENT Model / Volz et al, 2012)</i>	2D Exner equation	2D shallow water equations	Empirical bed-load transport formulas and advection-diffusion equations for suspended-load transport	Slope stability	<ul style="list-style-type: none"> Does not model seepage flow Needs very low time step 	?	Volz et al. (2012)
<i>Mizutani et al. (2013)</i>		2D shallow water equations	Non-equilibrium model framework	Two-dimensional slope stability		?	Mizutani et al. (2013)
<i>DL Breach Simplified</i>	YZ: Trapezoidal, rectangular or triangular	Weir flow for overtopping, orifice for IE	non-equilibrium sediment erosion	Slope stability	<ul style="list-style-type: none"> Predefined breach development Does not model seepage flow 	Free	Wu (2013)
<i>Guan et al. (2014)</i>		2D shallow water equations	Sediment transport equation	Two-dimensional slope stability	<ul style="list-style-type: none"> Results are dependant on calibration parameters, such as Manning's n and crit. slope angle. Does not model seepage flow 	?	Guan et al. (2014)
<i>DL Breach 3D</i>	Calculated from resulting bed change from suspended and bed-load equations	3D Reynolds-averaged Navier-Stokes equations using finite-volume method.	3D non-equilibrium transport equations of suspended-load and bed-load		<ul style="list-style-type: none"> Does not model slope stability Focuses more on flood routing rather than breach development Large computational requirements associated with running the model 	?	Marsooli and Wu (2014)
<i>Zhong et al. (2017)</i>		Weir formula		Limit equilibrium method for slope stability	<ul style="list-style-type: none"> Only considers headcut erosion 	?	Zhong et al. (2017)